A Capacitively Loaded Half-Wavelength Tapped-Stub Resonator

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***Abstract*— This paper examines a new resonator which is created by adding capacitive loading to a half-wavelength tapped-stub resonator. This capacitively loaded resonator has the property that both the Q and the resonant frequency can be set independently. This property is important because it allows this resonator to be used as either a tracking filter which maintains a constant bandwidth or as a filter which can vary its Q while maintaining a fixed resonant frequency. In this paper, equations are derived for choosing capacitance values that yield a desired resonant frequency and value of Q. Examples of using this resonator as both a fixed-frequency variable-Q filter and a constant-bandwidth tracking filter are provided. Theoretical results are verified by measurements.**

***Index Terms*—Bandpass filter, capacitively loaded, microstrip resonators, Q, tapped-stub resonators, tracking filters, transmission-line resonators.**

I. Introduction

IN THIS paper, the authors create a resonator by adding capacitive loading to a half-wavelength tapped-stub resonator. A tapped-stub resonator circuit is created by changing the connection point of a stub on the main transmission line. By changing the tapping point without changing the length of the stub, the Q of the resonator can be changed without changing the resonant frequency. The standard tapped-stub resonator is a quarter-wavelength end-to-end across the main line, with one of the shunt stubs shorted to ground and the other open. The half-wavelength tapped-stub resonator replaces the short on the shorted stub with a quarter-wavelength open section of transmission line. Thus, the half-wavelength tapped-stub resonator has two open stubs.

Using Richards' transform [1], capacitively loading the tapped-stub resonator can be achieved by replacing the open stubs, or parts of the stubs, with capacitors. Because the half-wavelength tapped-stub resonator has open stubs on both sides of the main transmission line, capacitive loading can be added to both sides of the resonator. In this configuration, adjusting the value of both capacitors allows one to independently change the effective length of the resonator and the tapping point. This allows one to independently set both the resonant frequency and the Q of the filter. Accordingly, if voltage-controlled capacitors are used, changing the resonant frequency and Q of the resonator is a matter of providing the capacitors with an appropriate voltage. Under this scenario, a tracking filter could be created, which maintains a constant bandwidth over the frequency range being swept. Likewise, a filter could be created, which maintains a constant resonant or center frequency, but has a variable Q.

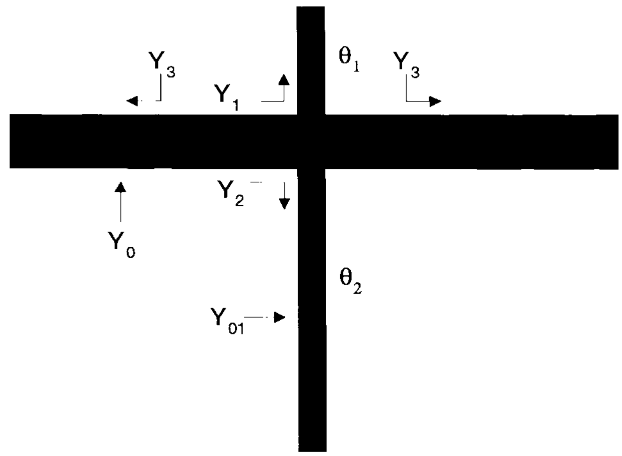

Fig. 1. Half-wavelength tapped-stub resonator.

The goal of this paper is to quantify the resonant condition and the Q of this filter. The authors begin by considering the half-wavelength tapped-stub resonator without capacitive loading. It is shown that by changing the tapping point of the resonator, one can change the Q without changing the resonant frequency. This leads to a discussion of the appropriate arrangement of capacitors which will give us adequate control of both the Q and the resonant frequency. The resonant frequency condition and the equation for Q for this capacitively loaded half-wavelength tapped-stub resonator is then derived, and these theoretical results are verified through measured data. Finally, it is demonstrated how the resonator can be used to vary bandwidth without changing the resonant frequency, and how it can be used as a constant bandwidth tracking filter.

II. Half-Wavelength Tapped-Stub Resonator

The standard tapped-stub resonator is a quarter-wavelength stub with one end open and one end shorted to ground [2]. The half-wavelength tapped-stub resonator replaces the short-circuit with an open quarter-wavelength section of transmission line, as shown in Fig. 1, where the total electrical

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length of the resonator is

$$\theta_1 + \theta_2 = \pi \frac{\omega}{\omega_0} = \pi \frac{f}{f_0}. \tag{1}$$

Introducing a tapping factor, k, the proportional length of the stub on each side of the main line, where $0 \leq k < 1$, is

$$\theta_1 = k \frac{\pi\omega}{2\omega_0} \tag{2}$$

and from (1)

$$\theta_2 = (2 - k) \frac{\pi\omega}{2\omega_0}. \tag{3}$$

Note that $k = 0$ represents an untapped half-wavelength open stub.

The Q of the half-wavelength tapped-stub resonator is obtained from the node admittance in Fig. 1:

$$Y = Y_1 + Y_2 + 2Y_3 = 2Y_0 + jY_{01}(\tan\,\theta_2 + \tan\,\theta_1) = G + jB. \tag{4}$$

Applying (4) to the fundamental definition of loaded Q [3]

$$Q = \left[\frac{\omega}{2G} \frac{\partial B}{\partial \omega}\right]_{\omega=\omega_0} \tag{5}$$

yields [4]

$$Q = \frac{\pi Y_{01}}{4Y_0} \sec^2 k \frac{\pi}{2} = \frac{\pi Y_{01}}{4Y_0 \cos^2 k \frac{\pi}{2}}. \tag{6}$$

From (6), note that as $k \to 0$, $Q \to \pi Y_{01}/4Y_0$, and as $k \to 1$, $Q \to \infty$. Thus, a wide range of frequency selectivity is possible.

III. Theory for the Capacitively Loaded Half-Wavelength Tapped-Stub

As is commonly known, the input admittance of an open-circuited section of transmission line is capacitive if the section is less than a quarter wavelength, $Y_{\text{in}} = jY_0 \tan(\omega l/v)$. Thus, capacitive loading of a tapped-stub resonator is created by replacing the stubs, or parts of the stubs, with capacitors. With the standard tapped circuit, one can replace the open stub with a capacitor or a series of capacitors. This arrangement has been demonstrated by DiPiazza *et al.* [5], [6].

If the half-wavelength tapped-stub resonator is used, there are two open stubs which can be replaced with capacitors. Thus, one can replace the shorter stub with a capacitor while simultaneously replacing the longer stub by a quarter-wavelength section of transmission line and a capacitor. The layout of this circuit is shown in Fig. 2. This design has three advantages over DiPiazza's arrangement. First and foremost, since this circuit has capacitors on both sides of the tapped stub, this circuit has an additional degree of freedom. This allows one to control both the resonant frequency and the Q simultaneously. Second, since the design is based on the half-wavelength tapped-stub resonator, the Q is twice that of the standard tapped stub and thus it is easier to obtain higher Q filters. Third, this layout leads to a more symmetrical frequency response.

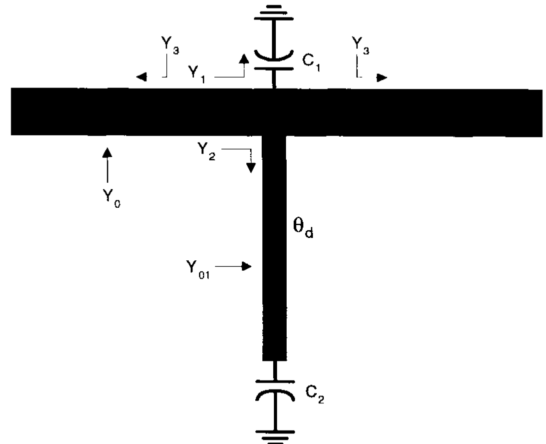

Fig. 2. Half-wavelength tapped-stub resonator with capacitive loading.

A. Deriving the Resonant Frequency

Referring to Fig. 2, the combination of C_1 and C_2 as well as the length of the quarter-wavelength section determines the total stub length which sets the resonant frequency of the circuit f_0. It is possible for the resonant frequency to be different from the frequency at which the quarter-wavelength transmission-line section is 90°. Therefore, it is necessary to introduce the design frequency f_d, which is the frequency at which this quarter-wavelength section is indeed a quarter wavelength.

The resonant frequency occurs at the point where C_1, C_2, and the quarter-wavelength section are effectively 180° in electrical length. Thus

$$\tan^{-1} \frac{\omega_0 C_1}{Y_{01}} + \tan^{-1} \frac{\omega_0 C_2}{Y_{01}} + \frac{\pi\omega_0}{2\omega_d} = \pi. \tag{7}$$

Solving (7) yields

$$\omega_0 Y_{01} \frac{C_1 + C_2}{Y_{01}^2 - \omega_0^2 C_1 C_2} = \tan\left(\pi - \frac{\pi f_0}{2f_d}\right) = -\tan \frac{\pi f_0}{2f_d}. \tag{8}$$

Equation (8) can also be rewritten as

$$\frac{Y_{01}^2 - 4\pi^2 {f_0}^2 C_1 C_2}{2\pi f_0 (C_1 + C_2)} = -Y_{01} \cot \frac{\pi f_0}{2f_d}. \tag{9}$$

Equation (9) yields the relationship between C_1 and C_2 for resonance to occur at the frequency f_0, given the design frequency f_d and a characteristic admittance Y_{01} for the quarter-wavelength section. Although (9) is fairly simple, there is no closed-form relationship between C_1 and C_2. A special case of (9) occurs when f_0 is the same as f_d. Under this condition, (9) simplifies to

$$f_0 = f_d = \frac{Y_{01}}{2\pi\sqrt{C_1 C_2}}. \tag{10}$$

B. Derivation of the Q

One finds the Q of the circuit shown in Fig. 2 from the total admittance at the common node to ground for the circuit shown in Fig. 2

$$\begin{aligned} Y &= Y_1 + Y_2 + 2Y_3 \\ &= 2Y_0 + j\left(\omega C_1 + Y_{01} \frac{\omega C_2 + Y_{01} \tan\theta_d}{Y_{01} - \omega C_2 \tan\theta_d}\right) \end{aligned} \tag{11}$$

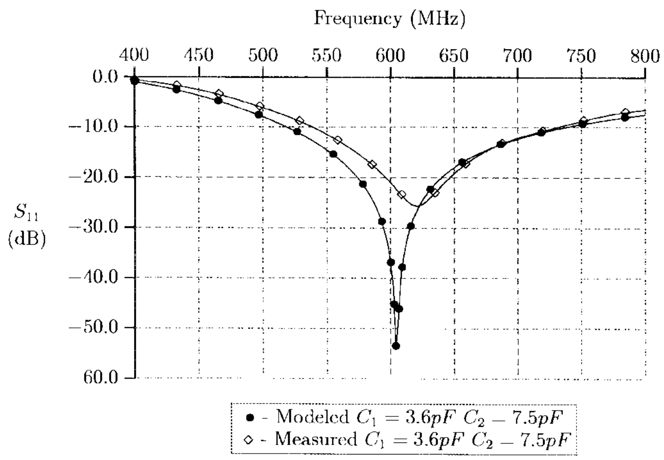

Fig. 3. Modeled and actual results for $C_1 = 3.6$ pF, $C_2 = 7.5$ pF.

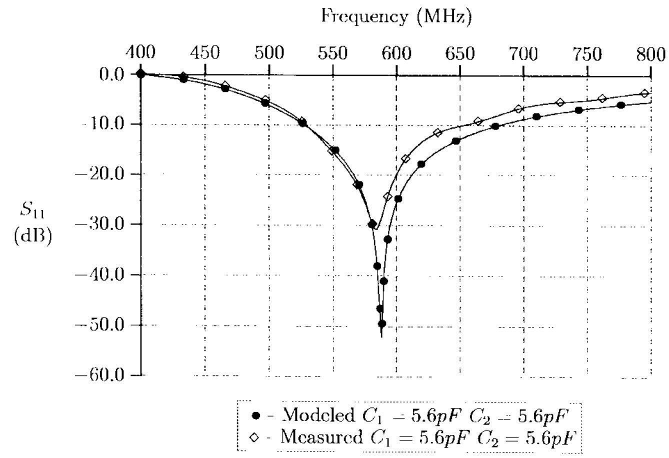

Fig. 4. Modeled and actual results for $C_1 = C_2 = 5.6$ pF.

where $\theta_d = \pi f / 2 f_d$. Using (5) and (11), the general expression for Q for this resonator is determined as shown in (12) at the bottom of the page. Note in (12) that increasing C_1 unambiguously results in a higher Q. Thus, increasing C_1 has the same effect as lengthening the shorter stub θ_1 in the tapped-stub layout of Fig. 1. The effect of C_2 is a little more complicated, but ultimately C_2^2 in the denominator dominates, and as C_2 gets larger, Q gets smaller.

In the case where the resonant frequency equals the design frequency $f_0 = f_d$, $\theta_d = \pi/2$ and (12) simplifies to

$$Q = \frac{\omega_0 C_1}{4Y_0} + \frac{\pi Y_{01}}{8Y_0} + \frac{Y_{01}^2}{4Y_0 \omega_0 C_2} + \frac{\pi Y_{01}^3}{8 Y_0 \omega_0^2 C_2^2}. \tag{13}$$

Combining (10) and (13) yields

$$Q = \frac{Y_{01}}{4Y_0}\sqrt{\frac{C_1}{C_2}} + \frac{\pi Y_{01}}{8Y_0} + \frac{Y_{01}}{4Y_0}\sqrt{\frac{C_1}{C_2}} + \frac{\pi Y_{01}}{8Y_0}\frac{C_1}{C_2} \tag{14}$$

which simplifies to

$$Q = \frac{\pi Y_{01}}{8Y_0}\left(1 + \frac{4}{\pi}\sqrt{\frac{C_1}{C_2}} + \frac{C_1}{C_2}\right). \tag{15}$$

Clearly, from (15), Q is controlled by the ratio of C_1 and C_2, while from (10) the center frequency f_0 is controlled by the product of C_1 and C_2. Thus, Q and f_0 may be controlled independently. Finally, note that in (15), as in (12), when C_1 increases, Q increases, and as C_2 increases, Q decreases.

IV. Results

A. Discussion of Data Collection

This section briefly describes the authors' data collection procedure for both modeled and measured results. Measured results were obtained from circuits constructed in microstrip using Rogers RT Duroid 5880 with a two-sided 1-oz rolled copper cladding. The relative dielectric constant was 2.2 and the substrate thickness was $\frac{1}{8}''$. The capacitors used were chip capacitors manufactured by Dielectric Laboratories, Inc. Each capacitor had a thickness of $\frac{1}{8}''$ and was mounted by drilling a hole through the dielectric substrate at the appropriate location. Electrical contact was made by soldering the capacitors in place. Measured data was collected using a Hewlett-Packard 8753A network analyzer. The network analyzer was properly calibrated, but no attempt was made to correct for connector losses or fixture parasitics. Modeled results were collected using a transmission line software package called "Puff." The modeled circuits were also in a microstrip format with a dielectric constant of 2.2 and substrate thickness of $\frac{1}{8}''$. Dielectric and connector losses were not modeled. Capacitors were also considered to be ideal with effectively no thickness or width. The Q values for both modeled and measured data were determined by a technique for measuring Q that uses S-parameter data [4].

B. Comparison of Measured and Modeled Results

To qualitatively compare the authors' theoretical results with measured and modeled results, three cases for the half-wavelength tapped-stub resonator with capacitive loading were created. Fig. 3 shows measured and modeled S_{11} data as a function of frequency using the capacitance values $C_1 = 3.6$ pF and $C_2 = 7.5$ pF. Fig. 4 shows the case where $C_1 = C_2 = 5.6$ pF, which corresponds to $\theta_1 = 45°$ in Fig. 1. Fig. 5 uses the capacitance values $C_1 = 8.2$ pF and $C_2 = 3.6$ pF. These capacitance values were chosen so that the circuit would have its first resonance point f_0 around 600 MHz, using a design frequency f_d of 600 MHz. In addition, the following parameters were used: $Z_0 = 50\ \Omega$ and $Z_{01} = 50\ \Omega$. Note that for all three of these cases there is excellent agreement between modeled and measured results. Discrepancies between modeled and measured results can be

$$Q = \frac{1}{4Y_0}\left[\omega_0 C_1 + \frac{Y_{01}^2 \omega_0 C_2 (1 + \tan^2 \theta_d) + (Y_{01}\omega_0^2 C_2^2 + Y_{01}^3)\theta_d \sec^2 \theta_d}{(Y_{01} - \omega_0 C_2 \tan \theta_d)^2}\right] \tag{12}$$

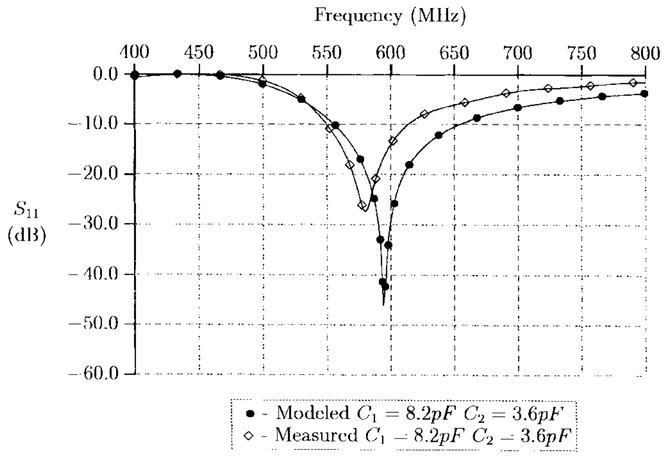

Fig. 5. Modeled and actual results for $C_1 = 8.2$ pF, $C_2 = 3.6$ pF.

primarily attributed to losses not accounted for in the model; namely, dielectric losses, fixture parasitics, capacitance losses, and connector losses. Also, the fact that all modeled lengths are exact, whereas measured lengths are only approximate, causes some disparity in the resonant frequency of measured results. Figs. 3–5 also demonstrate that as C_1 is increased while decreasing C_2, the Q of the filter progressively increases (bandwidth decreases) as implied by (15).

C. Fixed Frequency, Variable Q Filter

As mentioned above, the important property of this resonant structure is that one can independently set the Q and resonant frequency simply by choosing the appropriate values for C_1 and C_2. As a result, it is possible to change the Q of the resonator while leaving the resonant frequency unchanged by a judicious choice of C_1 and C_2. If variable capacitors are used for C_1 and C_2, a variable Q filter is created simply by applying the appropriate voltage, which yields capacitance values C_1 and C_2. The values of these voltages can be easily supplied by a look-up table based on the analysis provided in Section III.

An example of using this resonator as a fixed-frequency, variable-Q filter was created. Several cases with different values of C_1 and C_2 which yield the same resonant frequency were constructed. Resonant frequency f_0 of 600 MHz, and design frequency f_d of 600 MHz were chosen. Because $f_0 = f_d$, (10) and (15) were solved to yield the appropriate values of C_1 and C_2 for each desired value of Q. The results are shown in Fig. 6. These results are from actual circuits. In addition to using $f_0 = f_d = 600$ MHz, the following parameters were used: $Z_0 = 50\ \Omega$ and $Z_{01} = 50\ \Omega$. Table I lists the values of Q from theory, from modeled results, and from the actual circuits[1] for the cases shown in Fig. 6 [4]. Note that there is good agreement between theoretical, modeled, and actual Q values. From (15), increasing C_1 while decreasing C_2 will increase Q (and thus, decrease the bandwidth). This is indeed the case with $C_1 = 3.6$ pF, $C_2 = 7.5$ pF having the lowest Q and $C_1 = 18.0$ pF, $C_2 = 1.2$ pF having the highest Q. This result can be qualitatively observed in Fig. 6.

[1]Due to phase offset of the network analyzer, it was necessary to add an additional phase length to the phase data for each circuit.

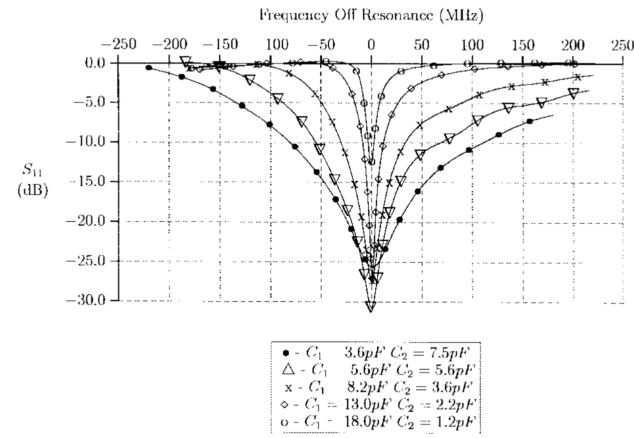

Fig. 6. Increasing Q with increasing C_1 (decreasing C_2).

TABLE I
Q FOR HALF-WAVELENGTH TAPPED-STUB RESONATOR WITH CAPACITIVE LOADING

Case	Theory	Modeled Circuit	Actual Circuit
$C_1 = 3.6pF, C_2 = 7.5pF$	.933	.923	1.02
$C_1 = C_2 = 5.6pF$	1.26	1.33	1.37
$C_1 = 8.2pF, C_2 = 3.6pF$	2.02	2.10	2.14
$C_1 = 13.0pF, C_2 = 2.2pF$	3.91	4.01	3.70
$C_1 = 18.0pF, C_2 = 1.2pF$	8.22	8.10	7.87

D. Constant Bandwidth Tracking Filter

Another benefit derived from being able to independently set the Q and resonant frequency is that it is possible to create a filter that both maintains a constant Q and changes its resonant frequency. The values of C_1 and C_2, which yield a given resonant frequency f_0 and Q, are obtained by simultaneously solving (9) and (12). In (9), as C_1 or C_2 is increased, f_0 is increased. In (12), increasing C_1 increases Q, while increasing C_2 decreases Q. These relationships guarantee that there will be an infinite number of resonant frequencies for a given Q by the choice of C_1 and C_2.

In practice, a constant-Q tracking filter is not as useful as a constant-bandwidth tracking filter. To make this conversion, the following relation between the 3-dB bandwidth and Q is used:

$$Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{\text{Bandwidth}} \tag{16}$$

where f_1 and f_2 are the lower and upper 3-dB points, respectively. By simultaneously equating (16) with (12) and using (9), the values of C_1 and C_2 which yield a desired resonant frequency for a specified bandwidth are solved. In this way, a lookup table is created which relates the choice of C_1 and C_2 to the resonant frequency f_0, given a desired bandwidth.

An example showing the operation of such a filter is presented. A bandwidth of 200 MHz with a fixed design frequency $f_d = 600$ MHz was chosen. Using a fixed design frequency implies that the quarter-wavelength section used

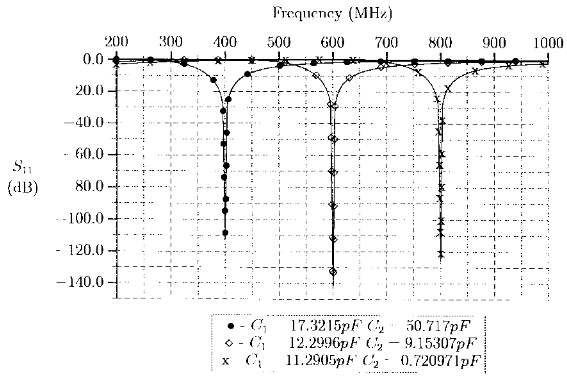

Fig. 7. Tracking filter which maintains a constant bandwidth.

does not change for different choices of resonant frequencies. This allows a constant bandwidth tracking filter to be created, which changes the resonant point by simply changing the voltage applied to the variable capacitors. In addition, the following parameters were used: $Z_0 = 50\ \Omega$ and $Z_{01} = 50\ \Omega$. We chose to sweep a range of frequencies from 400 to 800 MHz. Thus, three cases were created. The first uses capacitance values $C_1 = 17.3215$ pF, $C_2 = 50.7170$ pF, which yields a resonant frequency $f_0 = 400$ MHz. The second uses $C_1 = 12.2996$ pF, $C_2 = 9.1531$ pF, which yields $f_0 = 600$ MHz, and the third uses $C_1 = 11.2905$ pF, $C_2 = 0.7210$ pF, which resonates at $f_0 = 800$ MHz. Fig. 7 shows the results for these three choices. Note that even though the resonant frequency was swept over a range of 400 MHz, the 3-dB bandwidth remained constant at 200 MHz. Fig. 7 clearly indicates that using the capacitively loaded half-wavelength tapped-stub resonator to create an effective constant bandwidth tracking filter is very viable.

V. Conclusion

In this paper, it has been shown that the capacitively loaded half-wavelength tapped-stub resonator has the important property that both the Q and the resonant frequency of the filter can be set independently. Expressions were derived for the resonant frequency and the Q. It has been shown that these theoretical results agree very well with measured results. Furthermore, it has been demonstrated that by appropriate choices for the capacitance values, this resonator can vary the Q of the response without changing the resonant frequency. In addition, it has been shown how the capacitively loaded half-wavelength tapped-stub resonator can serve as a tracking filter that maintains a constant bandwidth over a wide range of frequencies.

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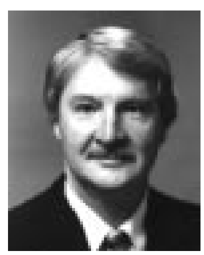

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